Code: CT22
Subject: DISCRETE MATHEMATICAL STRUCTURES
Time: 3 Hours
Max. Marks: 100

NOTE:
• Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.
• Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.
• Parts of a question should be answered at the same place.

Q.1
a. State one procedure to obtain the principal disjunctive and principal conjunctive normal forms of any predicate formula using an example.

b. What is a bipartite graph? What type of graph a tree and star are?

c. Prove that \( n+1 C_r = n C_{r-1} + n C_r \)

d. Show how many different words can be formed out of the letters of the word VARANASI?

e. Simplify the logical expression \( \overline{X} Y + \overline{X} Z + YZ + \overline{Y}Z\overline{W} \)

f. State and prove the Euler formula to detect the planarity of a graph.

g. What are the possible ways to combine two automatons? 

(7 x 4)

Q.2
a. If P, Q and R are three atomic variables, obtain the principal disjunctive normal form for \((P \rightarrow (Q \land R)) \lor (\neg P \rightarrow (Q \lor R))\)

(9)

b. State and prove the condition to find out if a given graph is an Euler graph.

(9)

Q.3
a. On a set \( S = \{1,2,3,4,5\} \), find the equivalence relation on \( S \), which generate the partition \( \{\{1,2\},\{3\},\{4,5\}\} \). Draw the graph of the relation.

(9)

b. Show that the maximum number of vertices in a binary tree of height \( h \) is \( 2^{h+1} - 1 \).

(9)

Q.4
a. Solve the recurrence relation \( a_n = 4 a_{n-1} - 4 a_{n-2} \), \( n \geq 2 \) with initial conditions, \( a_0 = 6 \) and \( a_1 = 8 \).

(9)

b. Define Boolean algebra. Prove that the power set of any set form a Boolean algebra.

(9)

Q.5
a. What is a spanning tree of a graph? How to obtain fundamental circuits, given a spanning tree?

(9)
b. What is a Hasse diagram? Draw the Hasse diagrams of the following sets under the partial ordering relation “divides” and indicate those which are totally ordered.
   (i) \{2, 6, 24\}
   (ii) \{1,2,3,6,12\}
   (iii) \{3,9,27,54\}

Q.6  
   a. Obtain the incidence and the adjacency matrix of the directed graph given below. Also, from the adjacency matrix, show that the graph is strongly connected or not?

   ![Directed Graph Image]

   b. Prove that if \((A, \leq)\) has a least element, then \((A, \leq)\) has a unique least element.

Q.7  
   a. Define a Deterministic and Nondeterministic Finite automaton and explain.

   b. Construct the finite automaton for the state transition table given below.