NOTE:
- Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.
- Parts of a question should be answered at the same place.
- All calculations should be up to three places of decimals.

Q.1  
  a. Find a root of the equation \( x^3 - 4x - 9 = 0 \) using the bisection method in 4 stages.
  
  b. Apply Gauss-Jordan method to solve the equations 
     \[
     \begin{align*}
     2x - 3y + 4z &= 13; \\
     3x + 4y + 5z &= 40.
     \end{align*}
     \]
  
  c. Find the eigenvalues of the matrix 
     \[
     A = \begin{pmatrix}
     2 & 0 & 1 \\
     0 & 2 & 0 \\
     1 & 0 & 2
     \end{pmatrix}
     \]
  
  d. Find the missing terms in the following table
     \[
     \begin{array}{c|c|c|c|c}
     x & 0 & 1 & 2 & 3 \\
     y & 1 & 2 & ? & 16
     \end{array}
     \]
  
  e. Obtain the Chebyshev linear polynomial approximation to the function \( f(x) = x^3 \) on \([0,1]\)
  
  f. Evaluate \( \Delta^2 \cos 2x \).
  
  g. A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the lines x = 0 & x = 1 and a curve through the points with the following co-ordinate
     (7 × 4)
     \[
     \begin{array}{c|c|c|c|c|c}
     x & 0.0 & 0.25 & 0.50 & 0.75 & 1.0 \\
     y & 1.0 & 0.9896 & 0.9589 & 0.9089 & 0.8415
     \end{array}
     \]
     Estimate the volume of the solid formed using Simpson’s rule.

Q.2  
  a. Using the Gauss-Seidel method, solve the system
     \[
     \begin{align*}
     20x + y - 2z &= 17; \\
     3x + 20y - z &= -18; \\
     2x - 3y + 20z &= 25.
     \end{align*}
     \]
  
  b. Solve the following matrix equation using LU decomposition method
     \[
     \begin{pmatrix}
     1 & 3 & 8 \\
     1 & 4 & 3 \\
     1 & 3 & 4
     \end{pmatrix}
     \begin{pmatrix}
     x_1 \\
     x_2 \\
     x_3
     \end{pmatrix}
     =
     \begin{pmatrix}
     1 \\
     6 \\
     4
     \end{pmatrix}
     \]
Q.3  
(a) Find the largest eigenvalue and its corresponding eigenvector of the following matrix using Power method.
\[
\begin{pmatrix}
4 & 1 & -1 \\
2 & 3 & -1 \\
-2 & 1 & 5
\end{pmatrix}
\]  
(b) Use the Givens method to find the eigenvalues of the tridiagonal matrix
\[
\begin{pmatrix}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{pmatrix}
\]

Q.4  
(a) Determine the interpolating polynomial that approximates to the function given in the following table using Lagrange’s formula and find f(0.5).

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>3</td>
<td>6</td>
<td>11</td>
<td>18</td>
<td>27</td>
</tr>
</tbody>
</table>

(b) Find a real root of the equation \(x^3 + x - 3 = 0\) which is close to 1.2, correct to four decimal places using Newton’s method.

Q.5  
(a) Given that
\[
x \\
y
\begin{array}{|c|c|c|c|c|c|c|}
\hline
x & 1.0 & 1.1 & 1.2 & 1.3 & 1.4 & 1.5 & 1.6 \\
\hline
\hline
\end{array}
\]

Find \(\frac{dy}{dx}\) and \(\frac{d^2y}{dx^2}\) at \(x = 1.1\) and \(1.6\) using difference formula.

(b) For the data given in the table, find the minimum value of \(y\).

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.205</td>
<td>0.240</td>
<td>0.259</td>
<td>0.262</td>
<td>0.250</td>
<td>0.224</td>
</tr>
</tbody>
</table>

Q.6  
(a) Evaluate \(\int_0^6 \frac{dx}{1 + x^2}\) using (i) Trapezoidal rule (ii) Simpson’s \(\frac{1}{3}\) rule (iii) Simpson’s \(\frac{3}{8}\) rule and compare the result with its actual value.

(b) Evaluate the integral \(I = \int_{-1}^{1} (1 - x^2)^{\frac{3}{2}} \cos x\) dx using Gauss-Legendre 3-point formula.
Q.7  

a. Employ Taylor’s method to obtain approximate value of y at x = 0.2 for the differential equation \( \frac{dy}{dx} = 2y + 3e^x \), \( y(0) = 0 \).  

\[ (8) \]

b. Using Runge-Kutta method of fourth order solve for \( y(0.1) \), \( y(0.2) \) given that \( \frac{dy}{dx} = \frac{y^2 - x^2}{x^2 + y^2} \), \( y(0) = 1 \).  

\[ (10) \]